Parameters Evaluation of Ant Colony Algorithm based on TSP

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Abstract

Ant colony algorithm is a kind of new ecological system algorithm used to solve complicated combinatorial optimization problems, and it is also a kind of heuristic evolutionary algorithm based on population, which belongs to a kind of random search algorithm, and is used to solve Traveling Salesman Problem (TSP). As you can see in the mathematical model of ant colony algorithm, the parameter space of the algorithm is large and the reasonable parameters’ setting helps to improve the convergence speed and global optimization ability of the algorithm. Influencing the parameters of the algorithm performance mainly have three aspects: pheromone heuristic factor $\alpha$, expected heuristic factor $\beta$ and pheromone evaporation coefficients $\rho$. This paper firstly analyzes how each parameter affects search capability and convergence speed of ant colony algorithm, and then chooses TSPLIB. c151 database based on TSP problem as an example to validate the relationship between the algorithm performance and parameters, lastly which are given the optimal selection range of each parameter and the best configuration.

Key words: Ant colony algorithm, Traveling Salesman Problem, Convergence speed

1. INTRODUCTION

Ant colony algorithm was to simulate the group behavior from real ants searching food in the nature by Italian scholar M. Dorigo as early as in 1992 (Dorigo and Luca 1997). The algorithm has the characteristics of distributed computing and strong robustness, easily is fused with other optimization algorithms such as genetic algorithm (GA), simulating degradation algorithm (SA), evolution rules (EP) and so on. It has intelligent searching, global optimization, robustness and positive feedback, so more and more domestic and international experts and scholars have paid more attention to it.

Ant colony algorithm has been successfully applied to the TSP and other combinatorial optimization problems (Dorigo and Luca 1997; Huang, Zhou and Wang 2003), such as figure coloring problem, vehicle routing problem and job-shop scheduling problem, etc. (Coloni and Dorigo, Maniezzo 1994).

However, the algorithm is easy to fall into local optimum, stagnation and long search time, etc. In order to avoid these problems, the paper analyzes three influencing parameters from the ant colony algorithm and simulated experiments by real examples and finally determines range of three parameters.

2. DESCRIPTION OF TSP

Traveling Salesman Problem (TSP) is a typical combinatorial optimization problem. Its description is as follows: given N cities, there is a traveling salesman from a particular city, visit each city once and only once after returning to the original departure city, hope to find the shortest tour path.

Available mathematical method description: assumption has a set of N city $C = \{c_1, c_2, \cdots, c_n\}$, the distance between the two cities for $d(c_i, c_j) \in R^+, c_i, c_j \in C (1 \leq i, j \leq N)$.

To make objective function $T_D = \sum_{i=1}^{N-1} d(c_{\Pi(i)}, c_{\Pi(i+1)}) + d(c_{\Pi(1)}, c_{\Pi(2)}, \cdots, c_{\Pi(N)})$ to the smallest city sequence $\{c_{\Pi(1)}, c_{\Pi(2)}, \cdots, c_{\Pi(N)}\}$, $c_{\Pi(1)}, c_{\Pi(2)}, \cdots, c_{\Pi(N)}$ is a full arrangement about $1, 2, \cdots, N$.

3. BASIC ANT COLONY ALGORITHM

3.1. Mathematical model

After Ant colony algorithm is put forward that is primarily used to solve TSP problem and it can represent all solving the shortest path problem. TSP problem is used to as a sample to elaborate the basic mathematical model of ant colony algorithm (AS). To easily describe, define the following variables:

$m$—the number of ants

$n$—the number of cities

$d_{ij}$—the distance between city land city $j$, $i,j=1,2,\cdots,n$.

$\tau_{ij}(t)$—the $t$ moment pheromone’s number on the path $(i,j)$, $i,j=1,2,\cdots,n$.

$\eta_{ij}$—path $(i, j)$ visibility, inspired by the degree from the city $i$ to the city $j$, $\eta_{ij} = \frac{1}{d_{ij}}$
\( \alpha \) — pheromone heuristic factor
\( \beta \) — expected heuristic factor
\( \rho \) — pheromone evaporation coefficients
\( \Delta \tau_{ij} \) — in the cycle path (i, j) increased the amount of information
\( \Delta \tau_{ij}^k \) — in the loop, ant K in the path (i, j) released information
\( p_{ij}^k(t) \) — at the moment of t, the probability of ant K from city i to city j
\( \text{tabu}_k \) — Tabu table, record the ant k current passes through the city, the next time it will no longer select the city from the table
\( \text{allowed}_k \) — \( \{1,2, \ldots, n\} \)-tabu, in any iteration, the ant k allows and visits to the cities
\( Q \) — the amount of pheromone on the path from Ants in the iteration

The AS path optimization algorithm includes two steps, one is the ant colony building path, the other is the pheromone updating.

3.2. Ant colony building path

The beginning moment of the algorithm, set is equal to the amount of information on each path, \( \tau_{ij}(0) = C (C \text{ as a constant}) \). The m ant randomly is on the n cities, updating tabu table of each ant at the same time, make its first element as the current city of ants. Then, each ant selects the next to move city according to the amount of information on the path and length of the path. Specifically, at t moment in the city i, the probability from the ant k transferring to the city j \( p_{ij}^k(t) \) as follows:

\[
p_{ij}^k(t) = \left\{ \begin{array}{ll}
\frac{\tau_{ij}^k \eta_{ij}^k(t)}{\sum_{n=1}^{\text{allowed}} \tau_{in}^k \eta_{in}^k(t)} & j \in \text{allowed}_k(1) \\
0 & \text{otherwise}
\end{array} \right.
\]

In formula (1), \( \alpha \) as pheromone heuristic factor, shows relatively important degree of the ant pheromone from selection path and reflects ants forward guidance from the pheromone of ant in the process of movement path. Bigger and bigger of \( \alpha \) value, the possibility of choice walks on the path in front of ants is larger. As the expected heuristic factor, shows relatively important degree of the path length to choose the path of the ant and reflects moving forward guidance from the ants path length in the process. Bigger and bigger of value, the possibility of the state transition will be an approach to the greedy rules. In the formula (1), We is not hard to find that the transition probability from the k ant proportional \( p_{ij}^k(t) \) is proportional to \( \tau_{ij}^k(t) \) to a determinate TSP problem. Meanwhile, the ants are always walking on the path of the shortest distance and the highest concentration from the pheromone.

3.3. Pheromone updating

After all ants have completed a travel (i.e., after travel in the n cities and return to the starting point) update pheromone of each side, according to the formula (2).

\[
\tau_{ij}(t + n) = (1 - \rho) \ast \tau_{ij}(t) + \Delta \tau_{ij}(2)
\]

\[
\Delta \tau_{ij} = \sum_{k=1}^{m} \Delta \tau_{ij}^k (3)
\]

In formula (2), \( \rho \) represents volatilization coefficient of pheromone on the path, \( \rho < 1 \). \( 1 - \rho \) represents residual coefficient of pheromone.

According to the different pheromone update strategy, M. Dorigointroduced three kinds algorithm Model of basic ant colony (Dorigo and Gambardella 1996; Stutzle and Hoos 1998): Ant-Cycle Model, Ant-Quantity Model and Ant-Density Model.

Between the three kinds of model, their difference is the local information and the whole information. In Ant-Quantity Model and Ant-Density Model, because the ants visit every city and update the pheromone on the path, they adopt the local information. However, in Ant-Cycle Model, after ants visit all cities and update the pheromone on the path. Ant-Cycle Model adopts the whole information of ant colony (Ma and Zhang, 2008; Min and Ge, Zhang, Liang, 2006; Wang and Xie, 2002)

Use the formula (4) in Ant-Cycle Model:

\[
\Delta \tau_{ij}^k = \left\{ \begin{array}{ll}
\frac{Q}{L_k} & \text{if edge}(i, j) \in T^k \\
0 & \text{otherwise}
\end{array} \right.
\]

A large number of experiments show that Ant-Cycle Model has good performance in solving TSP problem, so we usually use this Model.
3.4. Implementing steps of solving TSP

Solving TSP steps of Basic ant colony algorithm, as follows:
Step 1: Parameter’s initialization. \(m, \alpha, \beta, \rho, Q, t=0\) and \(Nc=0\). \(Nc\) max is the biggest loop number. Set the initial moment every path information \(\tau_{ij}(0) = C\) as a constant), \(\Delta \tau_{ij}(0) = 0\).

Step 2: m ants are randomly placed in the \(n\) cities, updating every ant tabu table at the same time, make the current city of ants as its first element.

Step 3: For each ant \(k\), according to the transition rules of random state (formula (1)), it is transferred to the next city \(j\). At the same time, the \(j\) in the ant \(k\) tabu table, so loop, until all the ants complete \(n\) city traveling.

Step 4: Calculate travel path length of each ant \(L_k(k=1,2,…,m)\), and record the current optimal traversal sequence and path length.

Step 5: According to formula (2)(3)(4) updating information on the every edge.

Step 6: For each edge \((i,j)\), \(\Delta \tau_{ij} = 0\), \(Nc = Nc + 1\).

Step 7: If \(Nc \leq Nc_{max}\), goto step 2, else goto step 8.

Step 8: Output the optimum solution of this experiment \(C_{min}\), \(C_{min} = \min \{L_k\}_{k=1}^{m}\), \(L_{k_{min}}\) is the minimum of \(m\) path length in every loop.

4. KEY PARAMETER ANALYSIS AND SIMULATION EXPERIMENTS

Through the mathematical model of ant colony algorithm, space of the algorithm parameter is large and reasonable parameters setting helps to improve the convergence speed and the global optimization ability of the algorithm. In the ant colony algorithm, influencing parameters of the algorithm performance mainly has three aspects: pheromone heuristic factor \(\alpha\), expected heuristic factor \(\beta\) and pheromone evaporation coefficients \(\rho\). This paper mainly studies the three parameters, firstly analyzes how each parameter affects searching capability and convergence speed of ant colony algorithm, and then to TSPLIB95 the repository TSP problem as an example to validate the relationship between algorithm performance and parameters, lastly given the optimal selection range of each parameter and the best configuration.

When we studies the influence of parameters to AS algorithm performance, wholly adopts the Ant-Cycle Model.

4.1. Pheromones heuristic factor \(\alpha\)

Pheromone heuristic factor \(\alpha\) shows relative important degree of pheromone \(\tau_{ij}(t)\) to select the path for the ant, not only reflects that the ants accumulate information in the process of sports on subsequent ants on the strength of the guiding role, but also reflect strength from the ants random factors in the search path. Greater of \(\alpha\) value, the accumulation of the amount of information for subsequent ants routing guidance is stronger and the likelihood of ants walked before the path choice is the greater. Then collaborative strength between ants is strengthened, accelerating the convergence speed. However, the search of randomness is abated and the search space is smaller. The algorithm easily falls into local optimum, thus reducing the global search ability of the algorithm. However, when the \(\alpha\) value is smaller, search speed is reduced. Although the search space of algorithm increases, it may make the algorithm into pure, endless random searching, and it will make it difficult to find the global optimal solution in the algorithm.

Through the experimental simulation method, we could determine that Pheromones heuristic factor \(\alpha\) in the ant colony algorithm affected performance of the algorithm and its application in the choice to the actual problem. In the TSPLIB95 database, the TSP as an example for experiments, set the default value: \(\rho=0.6\), \(\beta=3\), \(m=20\), \(Q=1000\), \(\alpha \in \{0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0\}\), \(Nc_{max}=1000\), for different values of \(\alpha\), run five times, solve the average and the optimal value. The experimental results are shown in the table 1, and figure 1. In the figure 1, each red dot represents an optimal value of the path, plotted by on the horizontal axis and the optimal value of the path on the vertical.

Through the theory calculation and simulation test results analysis, \(\alpha\) value is too big or too small that will affect the global search ability of the algorithm, \(\alpha\) the best value range \(\in [1.0, 3.0]\).

4.2. Expected heuristic factor \(\beta\)

Expected heuristic factor \(\beta\) expressed heuristic information \(\eta_{ij} (\eta_{ij} = \frac{1}{\delta_{ij}})\) and it is the relative important degree selecting the path to ants. It not only reflects the path length in the process of movement of ants on the strength of the guiding role, but also reflects the ant apriority in the path searching, the strength of the deterministic factors. The value of \(\beta\) is bigger, the greater the information of ants routing guidance, the more the ants at the local point on the greater the chance of local shortest path choice. It is to speed up the convergence
speed of the algorithm, but also narrow the ant searching space on the optimal path. As search randomness is receded, the algorithm easy to falls into the local optimum. And when the value $\beta$ is smaller, the algorithm searching is based on pheromone positive feedback , without the bias in the heuristic information, which makes the ants quickly fall in the local optimum, greatly reduces the global optimization ability of the algorithm.

Table 1: Pheromones heuristic factor affecting the performance of algorithm

<table>
<thead>
<tr>
<th>Pheromones Heuristic Factor $\alpha$</th>
<th>Average Value</th>
<th>Optimum Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>465.64</td>
<td>462.37</td>
</tr>
<tr>
<td>1.0</td>
<td>452.83</td>
<td>441.654</td>
</tr>
<tr>
<td>1.5</td>
<td>453.59</td>
<td>443.568</td>
</tr>
<tr>
<td>2.0</td>
<td>451.74</td>
<td>441.592</td>
</tr>
<tr>
<td>2.5</td>
<td>456.72</td>
<td>446.462</td>
</tr>
<tr>
<td>3.0</td>
<td>452.99</td>
<td>445.317</td>
</tr>
<tr>
<td>3.5</td>
<td>463.82</td>
<td>457.818</td>
</tr>
<tr>
<td>4.0</td>
<td>466.13</td>
<td>458.172</td>
</tr>
<tr>
<td>4.5</td>
<td>477.23</td>
<td>461.019</td>
</tr>
<tr>
<td>5.0</td>
<td>468.16</td>
<td>459.877</td>
</tr>
</tbody>
</table>

Figure 1. Relationship between the pheromones heuristic factor $\alpha$ and the optimal path length

Through the experimental simulation method, we can determine that Expected heuristic factor $\beta$ in ant colony algorithm affects performance of the algorithm and its application in the choice to the actual problem. In the TSPLIB95 database, eil51.TSP as an example for experiments, set the default value: $p=0.6, \alpha=1, m=20, Q=1000, \beta \in \{1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0, 5.5, 6.0\}, N_{\text{cmax}}=1000$, for different values of $\beta$, run five times, solve the average and the optimal value. The experimental results are shown in table 2, and figure 2. In the figure 2, Each red dot represents an optimal value of the path, plotted by $\beta$ on the horizontal axis and the optimal value of the path on the vertical.

Table 2: Expected heuristic factor affecting the performance of algorithm

<table>
<thead>
<tr>
<th>Expected heuristic factor $\beta$</th>
<th>Average Value</th>
<th>Optimum Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>478.52</td>
<td>474.688</td>
</tr>
<tr>
<td>1.5</td>
<td>462.74</td>
<td>459.522</td>
</tr>
<tr>
<td>2.0</td>
<td>452.74</td>
<td>446.081</td>
</tr>
<tr>
<td>2.5</td>
<td>455.83</td>
<td>448.015</td>
</tr>
<tr>
<td>3.0</td>
<td>451.69</td>
<td>445.816</td>
</tr>
<tr>
<td>3.5</td>
<td>460.74</td>
<td>457.818</td>
</tr>
<tr>
<td>4.0</td>
<td>453.60</td>
<td>446.082</td>
</tr>
<tr>
<td>4.5</td>
<td>459.01</td>
<td>450.316</td>
</tr>
<tr>
<td>5.0</td>
<td>468.15</td>
<td>463.37</td>
</tr>
<tr>
<td>5.5</td>
<td>466.29</td>
<td>460.747</td>
</tr>
<tr>
<td>6.0</td>
<td>475.38</td>
<td>473.503</td>
</tr>
</tbody>
</table>
Through the theory calculation and simulation test results analysis, β value is too big or too small that will affect the global searching ability of the algorithm. β the best value range ∈ [2.0, 4.0]. We have analyzed in front of the separately for two heuristic factor α and β given the best range of choice, but in actual operation, we may need to do the right match between both factors. Because the global optimization ability of the ant colony algorithm must have strong randomness in the ant colony searching process, the fast convergence performance of algorithm requires having a higher randomness in the ant colony search process. Thus, it is important for the reasonable collocation of both comprehensive to solve algorithm performance.

4.3. Pheromone volatility factor

In the ant colony algorithm in order to prevent the pheromone flooding heuristic information due to excessive accumulation the path searching heuristic information almost loses guidance function, so the algorithm is introduced in the pheromone volatility system. After the ants pass through the path, pheromone will gradually evaporate vary with the passage of time. We adopt pheromone volatilization coefficients \( \rho \) that is used to indicate the volatilization of pheromone. The size of the pha has a close relationship with the global searchability and convergence speed of the ant colony algorithm, so the appropriate selection of \( \rho \) value can guarantee the ant colony algorithm that find the global optimal solution in a reasonable amount of time.

Generally, in ant colony algorithm, the pheromone volatilization coefficients \( \rho \) ∈ [0,1], 1 – \( \rho \) express pheromone remaining degree that its value indirectly reflects the strength relationship between individual ants. Bigger of \( \rho \) value, more volatile pheromone on the path, residual pheromone too few will have smaller appeal to subsequent ants, so collaborative abate between ants, random and the global searching ability of the algorithm will be increased, but reduced convergence rate. Smaller of \( \rho \) value, bigger of the remaining coefficient 1 – \( \rho \), the pheromone had longer life, saved the pheromone trails will dominate. The search path is searched again likely. The algorithm is such positive feedback effect. While it is possible to speed up the convergence speed, but will reduce the random and global searching ability of the algorithm.

Through the experimental simulation method, we can determine that \( \rho \) volatile factors of pheromone in the ant colony algorithm effected performance of the algorithm and its application in the choice to the actual problem. In the TSPLIB95 database, ei151.TSP as an example for experiments, set the default value: \( \alpha = 1, \beta = 3, m=20, Q=1000, \rho \in [0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9], N_{c_{max}}=1000, \) for different values of \( \rho \), run five times, solve the average and the optimal value. The experimental results are shown in the table 3, and figure 3. In the figure 3, Each red dot represents an optimal value of the path, plotted by \( \rho \) on the horizontal axis and the optimal value of the path on the vertical.

<table>
<thead>
<tr>
<th>Pheromone volatility factor ( \rho )</th>
<th>Average Value</th>
<th>Optimum Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>474.76</td>
<td>468.962</td>
</tr>
<tr>
<td>0.2</td>
<td>472.59</td>
<td>466.567</td>
</tr>
<tr>
<td>0.3</td>
<td>468.23</td>
<td>462.965</td>
</tr>
<tr>
<td>0.4</td>
<td>455.69</td>
<td>450.695</td>
</tr>
<tr>
<td>0.5</td>
<td>449.57</td>
<td>439.819</td>
</tr>
<tr>
<td>0.6</td>
<td>451.77</td>
<td>440.573</td>
</tr>
<tr>
<td>0.7</td>
<td>454.61</td>
<td>443.091</td>
</tr>
<tr>
<td>0.8</td>
<td>454.15</td>
<td>442.580</td>
</tr>
<tr>
<td>0.9</td>
<td>453.70</td>
<td>445.829</td>
</tr>
</tbody>
</table>
Figure 3. Relationship between pheromone volatility factor ρ and the optimal path length

Through the theory calculation and simulation test results analysis, p value is more bigger, the global search ability of the algorithm is better. But the more value will reduce the rate of convergence of the algorithm. Finally, considering the global searching ability and convergence speed of algorithm, the optimum value p the best value range ∈[0.5, 0.8).

4. CONCLUSIONS

Because the ant colony algorithm easily fell into the local optimum solving TSP, the paper has deeply analyzed and studied three important parameters’ influence of the pheromone heuristic factor α, the expected heuristic factor β and pheromone evaporation coefficients ρ, that effected the global searching ability and searching speed. Combined with practical examples and simulated experiments concluded the best value range of three parameters: α ∈ [1.0, 3.0], β ∈ [2.0, 4.0], ρ ∈ [0.5, 0.8].

Acknowledgements

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REFERENCES


http://www.iwr.uni-heidelberg.de/groups/comopt/software/TSPLIB95/.


