An Improved Fast Fractal Image Compression Coding Method

Haibo Gao, Wenjuan Zeng, Jifeng Chen, Cheng Zhang

College of Information Science and Engineering, Hunan International Economics University, Changsha 410205, Hunan, China

Abstract
The main purpose of image compression is to use as many bits as possible to represent the source image by eliminating the image redundancy with acceptable restoration. Fractal image compression coding has achieved good results in the field of image compression due to its high compression ratio, fast decoding speed as well as independency between decoding image and resolution. However, there is a big problem in this method, i.e. long coding time because there are considerable searches of fractal blocks in the coding. Based on the theoretical foundation of fractal coding and fractal compression algorithm, this paper researches every step of this compression method, including block partition, block search and the representation of affine transformation coefficient, in order to come up with optimization and improvement strategies. The simulation result shows that the algorithm of this paper can achieve excellent effects in coding time and compression ratio while ensuring better image quality. The work on fractal coding in this paper has made certain contributions to the research of fractal coding.

Key words: Image Compression, Fractal Theory, Coding and Decoding.

1. INTRODUCTION
Image compression is aimed to use as many bytes as possible to represent and transmit the original big image and to have a better quality in the restored image. Image compression can relieve the burden of image storage and transmission so that fast transmission and real-time processing can be realized on the internet, therefore, it is quite necessary to investigate image compression methods. Fractal theory, a frontier discipline, believes that the nature is composed of fractals (Dimitrios and George, 2012). In the world, there are few temporary symmetric and balanced objects and states while there are numerous permanent asymmetric and unbalanced objects and states and fractal geometry is the geometry which describes the nature. As a new cognitive method for human beings to explore complicated things, fractal has practical application significance in the fields involving organization structures and morphology. The reason why fractal theory is effective in image compression is that fractal objects are quite universal in the natural world. They seem to have very complicated statistical characteristics and visual features, but they contain little information, which can be iterated with several simple determination rules (Srdjan and Sladjana, 2016; Pascal and Christian, 2015).

Fractal is the morphologic features to fill the space in the form of non-integer dimensions; in other words, fractal comes from a thinking theory. In 1973, B.B.Mandelbrot has come up with the idea of fractal dimension and fractal dimension for the first time. After the establishment of fractal geometry, it has attracted the attention of many subjects quickly because it is of great value in both the theory and practice (John and Glenn, 2016). On the other hand, fractal image coding method is a new-type image compression method which emerges and develops in the past ten years. It encodes the image into a group of contractive mapping and approximates the object to be encoded with the fixed point of the contractive mapping. The traditional image compression technology built upon the information theory can barely compresses such kind of images. As for fractal coding, only to encode several transformation rules can obtain a very high compression ratio. However, since the natural scenery different greatly, there are still a lot of issues to be further researched by the people in fractal compression. In the year of 1988, Barnsley firstly proposes a fractal image compression coding method which is realized with applied iterative function theory by using the global and local self-similarities of the image, performs compression coding on aerial image and obtain the compression ratio of 1000:1. However, its algorithm has rather great limitations and the main defect is that manual intervention is required in the coding process. Based on fractal iterated function, this paper investigates the theory, methods and implementation techniques of fractal image compression coding, explores its working mechanism, evaluates its ability, makes up for its flaws, designs and realizes highly-efficient image compression algorithm (Shiping and Liyun, 2014; Jeronimo and Higinio et al., 1969).

Firstly, this paper investigates the basic principles of image compression coding and analyzes and explains lossless compression and lossy compression. Then, it researches the theoretical foundation of fractal image compression and introduces the universal fractal image compression process. On the basis of the above research,
it elaborates the idea and steps of the improved algorithm of the paper. Finally, the experiment simulation of the paper proves that the method of this paper can shorten the time of fractal image coding while ensuring better subjective effects of the image.

2. IMAGE COMPRESSION CODING

The reason why the image can be compressed is that there is much information redundancy in image data. Image redundancy includes spatial redundancy, time redundancy, structural redundancy, knowledge redundancy and information entropy redundancy. Spatial redundancy refers to the correlation among pixel points; time redundancy is the redundancy between two consecutive frames, structural redundancy, structural redundancy means that there are strong texture structures in the regions, knowledge redundancy means that it has fixed structures, i.e. the head of a person, visual redundancy means that certain image distortion is invisible for human eyes and information entropy redundancy means that the amount of information per unit is larger than its entropy. The pixel points which form the image are greatly correlated to each other in both the horizontal and vertical directions. Therefore, the image data has plenty of redundancy. In other words, it has a huge compression potential. In this way, spatial compression or intra-frame compression can be used to reduce the spatial correlation between the neighboring pixels.

The task of image data compression is, without affecting or slightly affecting the image quality, to reduce as much image data as possible and to transform the input variable \( x \) into another variable \( y \) with a nonlinear transformation, e.g. \( y = g(x), x = g^{-1}(y) \), as indicated by Fig. 1.

![Figure 1. Compression and expansion of image](image)

Image data compression includes lossless compression and lossy compression. Lossless compression means that to reconstruct (or restore, decompress) the compressed data and obtain the same data as the previous, such as run-length encoding (RLE), delta modulation encoding (DM), Huffman encoding and Lempel-Ziv-Welch (LZW) encoding while by making use of the feature that human eyes are not sensitive to certain frequency components of the image, lossy compression uses some highly-efficient data compression algorithms with limited distortion and it allows a loss of certain information. It can be applied to the compression of image and noises because they usually contain more information than our visual system and auditory system can receive and some missing data will not lead to misunderstandings to the noises or the image, but it can greatly increase the compression ratio. The common lossy compression encoding techniques include predictive coding, transform coding, model-based coding, hybrid coding and so on and they are mainly used to compress such information as the image and noises. By combining the advantages of lossless and lossy compression, there emerges the third compression technology, namely hybrid compression. It strives for a balance among compression efficiency, compression ratio and fidelity and it includes JBIG, H261, JPEG, MPEG and wavelet (Shuyuan and Bing, 2015). The image compression coding and decoding process is shown in Fig.2.

According to the steps in Fig.2, the data after mapping transformation of the original image becomes data rate output after quantizer and entropy coder. Bit-plane coding is an effective technique which processes the bit planes of the image independently to reduce the redundancy among the pixels. It decomposes a pair of multi-level images into a series of binary images and compresses each binary image with several common binary image compression methods. Bit-plane coding has two steps: bit plane decompression and bit plane coding. For grey or colored image, if every pixel is represented with \( k \)-bit, take the 0 and 1 out in the same bits and form \( k \) N\( \times \)N binary images (Roberto and Vladimir, 2013). Refer every binary image as a bit plane. Lena image which uses the aforesaid lossless binary image compression technology to process bit plane is shown in Fig.3.
3. PROCEDURES OF FRACTAL IMAGE COMPRESSION ALGORITHM OF THIS PAPER

The basic theoretical foundation of fractal compression encoding includes packed transform, affine transform, iterated function system theorem and collage theorem. The fractal compression mainly considers the grayscale distribution of the image as well as the strategy to find the probability. Introduce Hausdorff metric into the image set of the real world and form a complete metric space. Every point is not only an image, but also a compact subset. Hausdorff metric space is considered the space where fractal is located and the distance between the fractals is also measured by this Hausdorff distance. The definition of affine transformation: the form of a transformation

\[ w(x_1, y_1) = (ax_1 + bx_2 + e, cx_1 + dx_2 + f) \]  

(1)

Here, \(a, b, c, d, e\), and \(f\) are all real numbers, which are affine transformation coefficients, and \(w\) is called two-dimensional affine transformation. If the linear part of the affine transformation is compressed, then the affine transformation is compressed. \(A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\) determines the rotation, migration and expansion, \(b = \begin{pmatrix} e \\ f \end{pmatrix}\) determines the amount of translation. In the plane, any affine transformation can be expressed as a combination of rotation, expansion, reflection and translation. In the rectangular coordinate system, it can be represented in the following form.
\[
\begin{pmatrix}
x' \\
y'
\end{pmatrix} = w \begin{pmatrix}
x \\
y
\end{pmatrix} = \begin{pmatrix}
a & b \\
c & d
\end{pmatrix} \begin{pmatrix}
x \\
y
\end{pmatrix} + \begin{pmatrix}
e \\
f
\end{pmatrix}
\]

(2)

\[
\begin{pmatrix}
a & b \\
c & d
\end{pmatrix} = \begin{pmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{pmatrix} \begin{pmatrix}
1 & \tan \alpha \\
0 & 1
\end{pmatrix} \begin{pmatrix}
l_1 \\
l_2
\end{pmatrix}
\]

(3)

For more complex graphics, may require a number of different affine transformations to achieve. The structure and shape of the graph are controlled by the affine transformation group \( \{ w_i \} \). Since the form of affine transformation is the same, so the different shapes depend on the coefficients of the affine transformation. If the original image and its transformation graph are already known, the affine transformation coefficient can be sought as well as three points in the original image and three points in the transformation graph can be confirmed (Vijayshri and Vaishali, 2016; Dongyu and Shaping et al., 2016).

The methods to directly calculate various transformation matrix coefficients of IFS can only applied to the images with local and global self-similarities. However, it is quite difficult to search the IFS of many images with the above-mentioned methods. But if it is possible to segment the entire image into several blocks and the IFS of these blocks are known, then image compression can also be realized. The method is to establish a fractal base to only store the corresponding IFS codes to the fractals. There is a theorem in fractal geometry, each interation function system defines a unique fractal graph, which is called the attractor of the iterated function system. An IFS is made up of a complete metric space and a group of contraction mappings. The probability of the affine transform is set as the following formula.

\[
p_i = \frac{\text{Area}(w_i(B_n))}{\text{Area}(B_n)} = \frac{|a d_i - b c_i|}{\sum_{i=1}^{N} |a d_i - b c_i|}
\]

(4)

In this formula, \( B_n \) is the segmented image block, make the energy of \( B_n \) as \( Q_n \).

\[
Q_n = \sum_{(i,j) \in B_n} f(i,j)
\]

(5)

\( f(i,j) \) is the image grayscale of point \((i,j)\) and the probability can be defined as follows.

\[
p_i = \frac{E(w_i(B_n))}{Q_n}
\]

(6)

Here, \( E(w_i(B_n)) \) is the energy of \( B_n \) after going through \( w_i \) transform. At this time, \( p_i \) can better reflect the information of the distribution of the internal image grayscale and it can also instruct the image reconstruction. For a class of compression mappings, the collage theorem provides a method for measuring the degree of approximation of a set with the corresponding invariant set. In order to find a IFS to set equal to its attractor and a given or similar, it must find a series of compression mapping, which combined with the given set given under the mapping set equal or close (Zhuang and Xingbao, 2015; Mambaye and Cristinel et al., 2013).

![Figure 4. Universal fractal image compression process](image)
paper uses such parameters as correlation coefficient and variance. In the search process, specific range block only matches the domain which meets the conditions so as to reduce the matching time and accelerate the coding speed (Jianji and Nanning et al, 2013). Fractal image compression process is shown in Fig.4.

Fractal image compression coding first segments the image to be processed into proper blocks and normally, every block has relatively visual self-similarities. Collage theorem will be used to seek the IFS coding of every block. Divide the original image into non-overlapping range blocks and overlapping domain blocks and calculate their standard deviations respectively. Set the standard gray-level deviation value \( \varepsilon \) and eliminate the flat domain blocks, the of which is smaller than \( \varepsilon \). Meanwhile, take the range blocks, the standard gray-level deviation is smaller than \( \varepsilon \) as the flat blocks and approximate the constant blocks, the gray-level value is equal to the mean value of range blocks. The specific procedures of the image compression coding are as follows.

1. Input the image. It includes the scaling \( S \) and the shifting \( O \) of the brightness, the starting point of affine transformation \((x, y)\) and the type of affine transformation \( dir \), rotate with \( \theta = dir \times \pi / 4 \) as the axis of symmetry.

2. Segment the image. Segment the original image into several blocks (marked as \( R_i \)) according to the resolution of \( B \times B \) pixels and form \( R \) block pool. There is no overlapping between any neighboring \( R \) blocks and the union set is the original image.

3. Constitute the codebook. Slide a \( 2B \times 2B \) window from left to right and from top to bottom at a step length of \( d \) in the original image and obtain \( D \) block pool. Average every \( D \) block with 4 neighboring pixel value and obtain \( B \times B \) pixel block. The entire sub-block constitutes the codebook \( \Omega \).

4. In searching an optimal domain block in the range block, to match the existing \( R \) block is to search the domain block with as small \( MSE \) of the existing \( R \) block as possible in order to obtain the fractal code of the existing \( R \) block. The closer to 1 the absolute value of the correlation coefficient is, the more matching these two blocks are. Given a domain block set \( D \) and its corresponding transformation \( \omega \), \( \lambda = \min |R-\omega(D)| \) goes to 0 for the optimal domain block \( R \).

5. The copy of every block must be smaller than the original block so as to guarantee the contractility of affine transformation. Encode every \( R \) block and repeat Step 3 on all \( R \) blocks. It is better that the copies used in collage without overlapping or vacancy. Set the probability \( p_i \) of affine transformation and ensure that \( \sum_{i=1}^{N} p_i = 1 \) works. Output the image fractal code after quantization and get the fractal image compression coding document (Ching and Oi et al, 2013; S.A. and A.W.C. et al, 2013).

4. ANALYSIS OF EXPERIMENT SIMULATION

4.1. Objective Measurement Index of Image Compression Coding

Image compression coding is generally represented with the following indexes.

1. CR (Compression Ratio)

\[
CR = \frac{B_f}{B_s}
\]

Here, \( B \) refers to the number of bits included before image compression and \( B_s \) is the number of bits after image compression. The bits of every pixel is also called bit rate with a unit of bit/s and it is an important performance index to describe compression technology or system.

2. PSNR (Peak Signal to Noise Ratio)

\[
PSNR = 10 \log_{10} \left( \frac{(2^n - 1)^2}{MSE} \right)
\]

Here, \( n \) refers the number of bits per pixel, and \( (2^n - 1) \) represents the levels of quantization of image gray-level value. \( MSE \) refers to the mean square error after normalization processing with its definition form as follows.

\[
MSE = \frac{1}{M \times N} \sum_{i=1}^{M} \sum_{j=1}^{N} (X(i,j) - Y(i,j))^2
\]

Here, \( M \times N \) refers to the image size, \( M \) and \( N \) are the height and width, \( X(i,j) \) & \( Y(i,j) \) represent the gray level of the original image and the reconstructed image at the point of \((i, j)\),

4.2. Image Compression Result & Analysis
In fractal image compression, the fractal size reflects the size of the search space in the matching process. The smaller the size is, the bigger the search space form is and the longer the corresponding encoding time is. However, in order to guarantee the decoding quality of the image, the image must be divided into small blocks so as to make sure that the matching blocks with many similarities can be found. In this experiment, the size of the range domain is $4 \times 4$, the size of the domain block is $8 \times 8$, the gray standard deviation $\sigma = 3$. The algorithm is realized with Matlab programing language, the operation platform is Windows 7 and the processor is Intel(r) Core(tm) i3, 2.40GHZ. The encoding performance will be measured with the encoding time(s) and the objective measurement of image quality PSNR (dB). Perform compression simulation on the standard images of Onion and Cameraman with the algorithm of this paper and the simulation results are shown in Fig. 5 and Fig. 6 as well as Table 1 and Table 2.

<table>
<thead>
<tr>
<th>Method</th>
<th>PSNR(dB)</th>
<th>MSE</th>
<th>Encoding time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traditional fractal algorithm</td>
<td>32.2538</td>
<td>$1.6130e+007$</td>
<td>48.54</td>
</tr>
<tr>
<td>Algorithm of this paper</td>
<td>31.8986</td>
<td>$1.5991e+007$</td>
<td>39.38</td>
</tr>
</tbody>
</table>

Table 1. Comparison of image reconstruction parameters of Onion

<table>
<thead>
<tr>
<th>Method</th>
<th>PSNR(dB)</th>
<th>MSE</th>
<th>Encoding time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traditional fractal algorithm</td>
<td>30.3705</td>
<td>$1.5622e+007$</td>
<td>57.03</td>
</tr>
<tr>
<td>Algorithm of this paper</td>
<td>30.0724</td>
<td>$1.5775e+007$</td>
<td>43.66</td>
</tr>
</tbody>
</table>

Table 2. Comparison of image reconstruction parameters of Cameraman

The above experiment data has proven that the algorithm of this paper can have better PSNR and better subjective compression effects. Compared with the traditional fractal coding algorithm, the decoding image by algorithm of this paper has degraded, but its encoding speed has been accelerated. Therefore, the fractal compression coding method of this paper has not only fully combined the advantages of the traditional fractal coding algorithm, but it has also avoided the defects of the traditional algorithms so that it has made a big step forward.
5. CONCLUSIONS

Image compression is an efficient way to solve the problem of big storage and transmission data of digital image. After mainly studying the fractal-based compression methods, the paper has proposed an improved fast fractal image compression coding method. This method optimizes the search process by using the relationship between the correlation coefficient and variance between the range block and its matching definition domain block so as to accelerate the matching speed. In the meanwhile, it takes the texture features of the image into consideration and guarantees the quality of the reconstructed image. In the end, this paper has analyzed and verified the algorithm of this paper through experiment and the result shows that the method of this paper is quite effective.

ACKNOWLEDGEMENT

This work was supported by Hunan Provincial Education Science five-year plan funded project (No: XJK014BGD046), Scientific Research Fund of Hunan Provincial Education Department in 2015 (No: 15C0780, No: 15C0779).

REFERENCES


